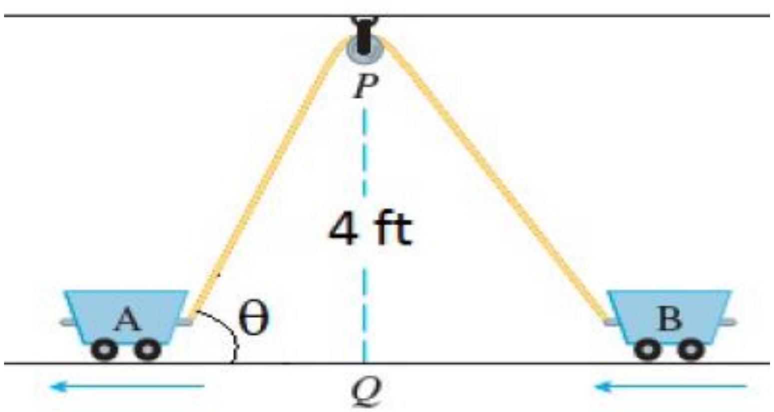
Closing *Thurs*: 3.9
Closing next *Tues*: 3.10
Closing next *Thurs*: 4.1(1), 4.1(2)
Remember: Friday is a Holiday (no class)

Entry Task (2016 Midterm Question):
Two carts are connected by a rope 15 ft long that passes over a pulley *P*. The point *Q* is on the floor 4 ft beneath *P*. Cart *A* is being pulled away from *Q* at a constant speed of 2 ft/s.

(a) Find the rate at which θ is changing when cart *A* is 3 ft from *Q*.

(b) How fast is cart B moving when cart *A* is 3 ft from *Q*?



3.10 Linear Approximation

Idea: "Near" the point $(a, f(a))$ the graphs of $y = f(x)$ and the tangent line

$$y = f'(a)(x - a) + f(a)$$

are close together.

We say the tangent line is a **linear approximation** (or **linearization** or **tangent line approximation**) to the function. Sometimes it is written as

$$L(x) = f'(a)(x - a) + f(a)$$

In other words:

If $x \approx a$,

then $f(x) \approx f'(a)(x - a) + f(a)$

Example: Find the linear approximation of $f(x) = \sqrt{x}$ at $x = 81$. Then use it to approximate the value of $\sqrt{82}$.

Example: Find the linearization of $g(x) = \sin(x)$ at $x = 0$. Then use it to approximate the value of sin(0.03).

Example:

Using tangent line approximation

estimate the value of $\sqrt[3]{8.5}$.

Example: (Newton's Method)

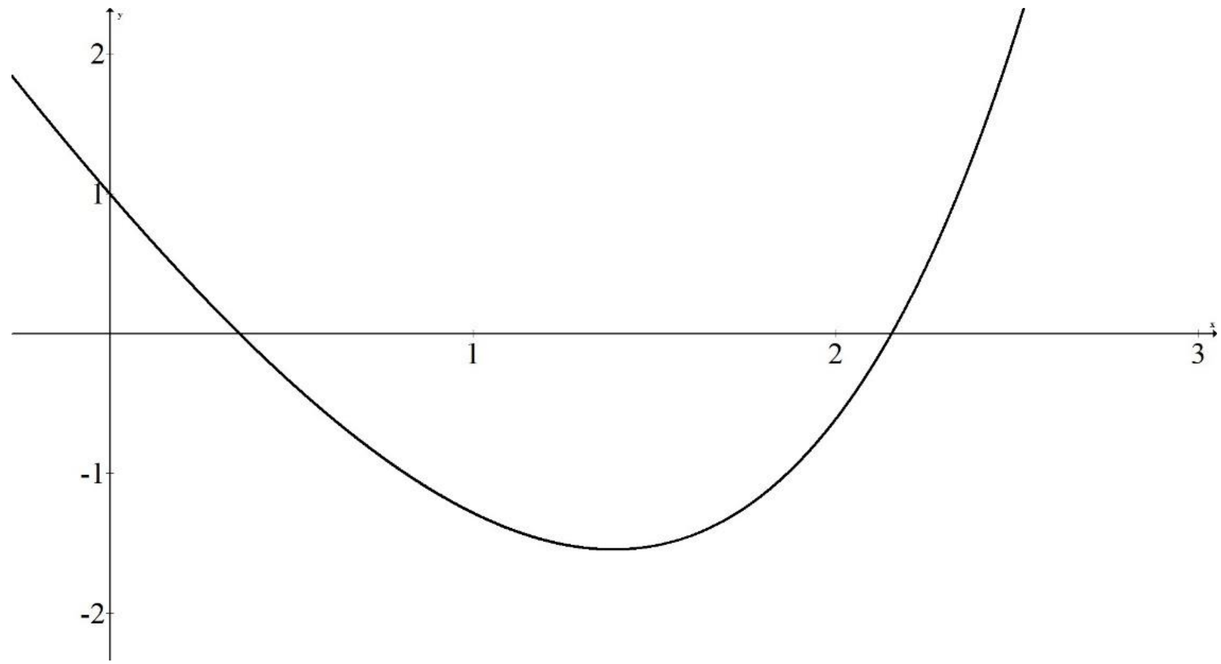
Consider $f(x) = e^x - 4x$.

From the graph, note that

$$e^x - 4x = 0$$

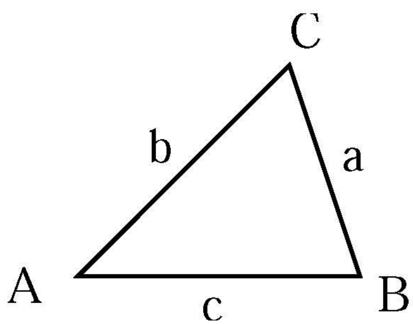
has one solution "near" $x = 0$.

Find the linearization at x=0, and use it to approximation this solution.



Some Homework Hints:

Problem 10: Suppose that *a* and *b* are pieces of metal which are hinged at *C*.



By the ``law of sines,'' you *always* have:

$$\frac{b}{a} = \frac{\sin(B)}{\sin(A)}$$

At first: angle *A* is $\pi/4$ radians= 45^o and
angle *B* is $\pi/3$ radians = 60^o.

You then widen *A* to 46^o, without changing the sides *a* and *b*.

The question asks you to use the linear approximation to estimate the new angle B.

Problem 8:
A cone with height *h* and base radius *r* has total surface area *S* given by:

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

You start with *h* = *8* and *r* = *6*, and you want to change the dimensions in such a way that the total *surface area remains constant*.

Suppose the height increase by 26/100.

In this problem, use tangent line approximation to estimate the new value of *r* so that the new cone has the same total surface area.